A Hybrid Technique Combining the Method of Moments in the Time Domain and FDTD

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***Abstract*— This letter presents a new hybrid method that efficiently combines two versatile numerical techniques, viz., the finite difference time domain (FDTD) and the method of moments in the time domain (MoMTD). The hybrid method is applicable to complex geometries comprising arbitrary thin-wire and inhomogeneous dielectric structures. It employs the equivalence theorem to separate the original problem into two subproblems: 1) the region containing the wires, which is analyzed by using the MoMTD, and 2) the dielectric zone that is modeled with the FDTD. The application of the method is illustrated by analyzing two canonical problems involving thin wires and inhomogeneous media.**

***Index Terms*— FDTD, hybrid methods, method of moments, thin wires, time domain.**

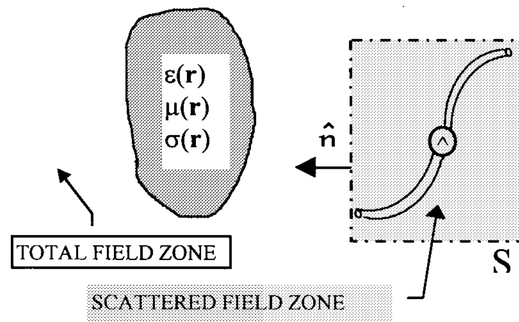

Fig. 1. Thin-wire antenna in the vicinity of a dielectric scatterer. FDTD computational domain.

I. Introduction

HYBRID methods, which combine the desirable features of two or more different techniques, are developed to analyze complex electromagnetic problems that cannot be resolved conveniently, and/or accurately, by using them individually [1]. An example of such a problem is an arbitrarily oriented thin-wire antenna located in the vicinity of an inhomogeneous dielectric scatterer (Fig. 1), for which a convenient, efficient, and accurate modeling technique is needed. The finite-difference time-domain method (FDTD), which is based on the direct solution of Maxwell's curl equations, is ideal for treating the inhomogeneous region in this problem; its capability for dealing with complex geometries with arbitrary electrical properties is well known [2]. However, the application of the FDTD to a thin wire, which, in general, is curved and arbitrarily oriented, presents some difficulties. To model the thin wire features precisely, a drastic reduction in the cell size of the FDTD grid must be employed. This, in turn, introduces a substantial increase in computation time and memory requirements if a uniform mesh is used to model the entire computational domain. Although an alternative might be to use a nonuniform mesh or a local subcell approach, these are still susceptible to errors [3] introduced by the staircasing. Furthermore, they may, in fact, be less accurate than their uniform counterpart, or even may suffer from instability problems [4]. Yet another possibility is to employ a conformal mesh, though this may require complicated preprocessing or modification of the update equations. In contrast, the integral equation methods are well-suited for the study of thin wires embedded in a homogeneous environment. In particular, the electric field integral equation (EFIE), derived by enforcing the boundary condition on the tangential electric field over the surface of the wires, has been widely used for analyzing thin-wire structures, both in the frequency as well as directly in the time domain [5].

Problems involving combinations of thin-wire antennas and arbitrary dielectrics arise in numerous applications, including ground-penetrating radar, mobile communication, and electromagnetic stimulation of biological tissues. Huang *et al.* [6] have recently introduced a hybrid method that employs the method of moments (MoM) in the frequency domain to solve the thin-wire antenna problem and the FDTD method to handle the inhomogeneous dielectric object. This approach employs a combination of Fourier transformation and iteration, and exchanges information on the field values, back and forth, between the antenna subproblem and the dielectric inhomogeneity region. The iterative technique is rapidly convergent only when the mutual interaction between the antenna and the dielectric body is relatively weak.

In this letter, we consider the transient excitation of a thin-wire antenna located in the proximity of an inhomogeneous dielectric scatterer, and we propose a new hybrid method that combines the FDTD algorithm to analyze the inhomogeneous dielectric part of the configuration, with the MoM in the time domain (MoMTD) to model the thin-wire antenna as described

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in [5], [7], and [8]. Since the method is implemented entirely in the time domain, it can efficiently generate information over a wide frequency band. Furthermore, the method does not require an iterative procedure to model the coupling between different regions of the problem, but employs a time-stepping procedure in a recursive manner to calculate the desired response by invoking the causality principle.

II. Decription of the Hybrid Method

The hybridization technique is based upon the use of the surface equivalence theorem (Huygen's principle). It begins by dividing the original problem into two separate ones. The first one of these, which contains the thin-wire structure, is solved by using the MoMTD as described in [5], [7], and [8], while the second, which deals with the dielectric materials, is handled via the FDTD scheme. A time-stepping solution procedure is implemented as follows.

1) A Huygen's surface S is introduced around the thin-wire antenna (Fig. 1). The equivalent electric and magnetic sources on S are deduced at each time step from the electric and magnetic fields created by the antenna in free space. These fields are obtained from the currents on the antenna that have been previously calculated by solving the EFIE via the MoMTD approach. The equivalent sources on S, when located in free space, produce the correct incident fields outside the Huygen's surface and zero fields inside it.
2) Next, the FDTD algorithm is applied in the entire computational domain, except with the antenna removed (it has been replaced by the equivalent sources). The equivalence principle is implemented in the 3D-FDTD computer code by applying the procedure given in [9], which does not require an averaging of the magnitudes involved. The field scattered by the dielectric material is computed inside S (scattered field zone) and the sum of the incident and the scattered fields is calculated outside S (total field zone). In the absence of the wire antenna, the FDTD solution inside S is, by definition, the incident field on this antenna.
3) At each time step, the currents induced on the antenna are calculated by solving the EFIE via the MoMTD approach from the values of the currents at previous time steps, and from the incident field on the antenna at the present time which has been computed by using FDTD as described above. Linear interpolation in the spatial and temporal domains is applied to calculate this field at specific locations on the antenna and at specific time steps.

III. Numerical Examples

To validate the technique, the hybrid method was first applied to the simple case of a thin, bent-wire antenna located in the proximity of a perfectly electric conducting (PEC) ground plane, as shown in the inset in Fig. 2. The length of each arm of the wire is 0.2 m, the radius 2 mm, and the distance to the PEC plane is 0.5 m. The antenna is modeled with eight segments and excited at its center by an 8-ns Gaussian source

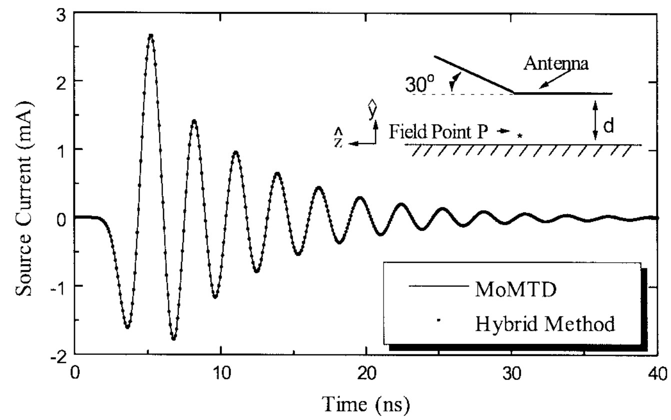

Fig. 2. Source current versus time for a bent-wire antenna above a PEC ground plane.

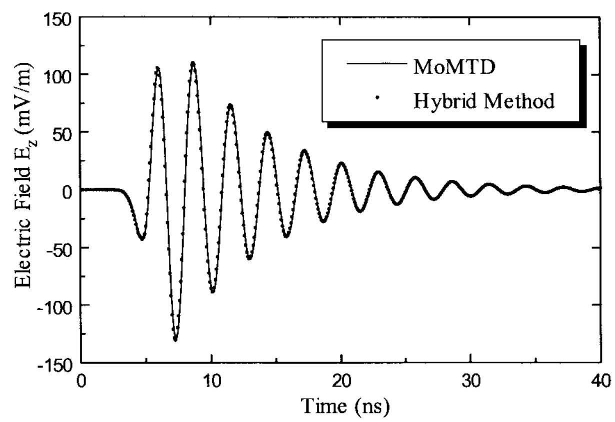

Fig. 3. Bent-wire antenna above a PEC ground plane: z-component of the total electric field at a point 5 cm from the ground plane.

voltage with unit amplitude. For the FDTD solution a cell size of 5 cm is chosen ($\lambda/12$ at the highest frequency). The dotted line in Fig. 2 shows the time-domain signature of the current at the feed point of the antenna, obtained by applying the proposed hybrid method. Also included in this figure (solid line) is the current at the same point for the equivalent configuration of two bent wires that are mirror images of each other. The latter configuration has been analyzed by the MoMTD code DOTIG1 [5]. Fig. 3 shows the z-component of the electric field E, calculated via both of the methods described above, at a point P, which is located 5 cm from the ground plane. The results obtained with the hybrid scheme are in very good agreement with those derived via the MoMTD approach.

Another example considered is a straight thin-wire antenna 0.4 m in length, located 0.5 m above a homogeneous half-space whose relative dielectric permittivity is $\varepsilon_r = 4$ (see the inset in Fig. 4). The radius of the antenna is 2 mm and it is excited at its center by the same Gaussian source voltage described in the previous example. The wire is modeled with eight segments and the cell size for the FDTD computation is 2.5 cm. The results obtained from applying the hybrid method were compared with those calculated by the MoM [10]. The current at the feed point obtained with both methods is plotted in Fig. 4. Fig. 5 shows the z-component of the

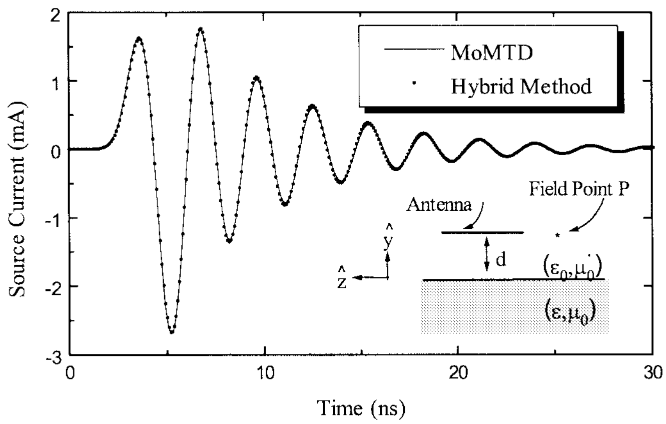

Fig. 4. Source current versus time for a straight-wire antenna above a dielectric half-space.

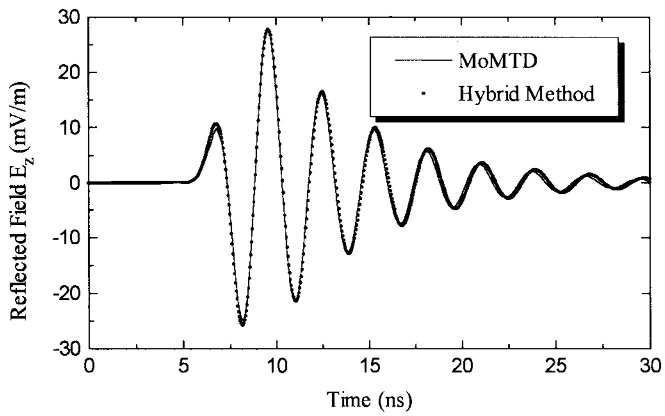

Fig. 5. Straight-wire antenna above a dielectric half-space: z-component of the electric field reflected at a point 0.2 m from the antenna.

electric field at the observation point P located at a distance 0.2 m from the wire (as P is located inside the Huygens surface, the magnitude represented is just the field reflected by the dielectric medium). Again, the agreement between the results obtained by the two techniques is excellent.

A complete stability analysis is beyond the scope of this letter; it should be mentioned, however, that the results were found to be numerically stable, even up to late times, in the examples considered above.

IV. Conclusions

We have described a new time-domain hybrid method, based on the surface equivalence principle, that combines the advantages of the MoM for studying arbitrary thin-wire structures, and of the FDTD for treating inhomogeneous dielectric bodies. It can be efficiently applied to investigate the transient excitation of structures formed by both kinds of geometries without using a conformal mesh technique, nonuniform gridding, or subcell modeling.

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